28) A roller coaster starts with a speed of 8.0 m/s at a point 45 m above the bottom of a dip (see figure). Neglecting friction, what will be the speed of the roller coaster at the top of the next slope, which is 30 m above the bottom of the dip?

![Image of roller coaster]

A) 8 m/s  
B) 12 m/s  
C) 17 m/s  
D) 19 m/s  
E) 21 m/s

**Solution**  
Only gravity is doing work. Since gravity is a conservative force mechanical energy is conserved:

\[
E_i = E_f \Rightarrow \frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f \Rightarrow v_f = \sqrt{v_i^2 + 2g(h_i - h_f)} \Rightarrow
\]

\[
v_f = \sqrt{(8.0\text{ m/s})^2 + 2 \cdot (9.8\text{ m/s}^2)(45\text{ m} - 30\text{ m})} \Rightarrow v_f = 19\text{ m/s}
\]

29) A block with mass \( m = 2.0 \text{ kg} \) is released from rest at the top of an incline. The incline makes an angle of 20° with the horizontal. The coefficient of kinetic friction between the block and the incline is 0.20. What will be the speed of the block after sliding 4.0 m along the incline?

A) 2.2 m/s  
B) 3.0 m/s  
C) 3.5 m/s  
D) 5.2 m/s  
E) 6.0 m/s

**Solution:**

\( h = d \sin \theta \)

\( N = mg \cos \theta \)

\( f_k = \mu_kN = \mu_kmg \cos \theta \)
Method 1 (using Newton’s Second Law):
\[ a = \frac{(mg \sin \theta - f_k)}{m} = g(\sin \theta - \mu_k \cos \theta) \]

\[ v_f^2 - v_i^2 = 2ad \Rightarrow v_f = \sqrt{v_i^2 + 2ad} \]

\[ v_f = \sqrt{2 \cdot (9.8m/s^2) (4.0m) (\sin 20^\circ - 0.20 \cos 20^\circ)} \]

\[ v_f = 3.5m/s \]

Method 2 (using conservation of energy):

\[ E_f - E_i = W_{\text{friction}} \Rightarrow \frac{1}{2} mv^2 - mgh = -f_k d \Rightarrow \frac{1}{2} mv^2 - mgd \sin \theta = -\mu_k mg \cos \theta d \]

\[ v_f = \sqrt{2gd(\sin \theta - \mu_k \cos \theta)} = 3.5m/s \]

30) A 1.00-kg block is on a frictionless surface and originally at rest. A 10.0-g bullet moving at 300 m/s is fired into the block. The bullet emerges on the opposite side of the block with half the bullet’s original speed and the direction of the original velocity. What is the speed of the block after the collision?

A) 1.50 m/s
B) 2.97 m/s
C) 15.0 m/s
D) 273 m/s
E) 300 m/s

**Solution:**

Conservation of linear momentum:

\[ mv = Mv_b + m\left(\frac{1}{2}v\right) \Rightarrow v_b = v \frac{m}{2M} \Rightarrow v_b = (300m/s) \frac{10.0 \cdot 10^{-3}kg}{2 \cdot 1.00kg} \Rightarrow v_b = 1.50m/s \]
31) The graph below shows the potential energy of a 1-kg particle confined to move along the \( x \)-axis as a function of position. The particle is moving to the right at point \( O \) with a kinetic energy \( K = 20 \text{J} \). Which of the labeled points is a turning point for the motion of the particle?

A) Point A  
B) Point B  
C) Point C  
D) Point D  
E) Point E

**Solution:**
At point \( O \), the total mechanical energy is \( E = U_O + KE_O = 15J + 20J = 35J \). At a turning point the kinetic energy is zero and the potential energy is equal to the total mechanical energy. At point \( E \): \( U_E = 35J \Rightarrow KE_E = E - U_E = 0 \Rightarrow \) Point \( E \) is a turning point.

32) The cross section of a piece of metal of uniform density is shown in the figure below. The \( y \)-position of the center of mass of the piece is at ______.

A) 0.7 m  
B) 0.8 m  
C) 0.9 m  
D) 1.0 m  
E) 1.1 m

**Solution:**
Let \( M \) be the mass of one square of the metal piece and let \( y_n \) be the \( y \) coordinate of the center of \( n \)'s square, then
\[
y_{cm} = \frac{\sum M y_n}{\sum M} = \frac{\sum y_n}{10} = \frac{2}{10} \left( -\frac{1}{2} + \frac{1}{2} + \frac{3}{2} + \frac{3}{2} + \frac{5}{2} \right) \text{m} = 1.1 \text{m}
\]
33) If a solid cylinder rolls without slipping, the ratio of its rotational kinetic energy to its translational kinetic energy is $E_{\text{rot}} / E_{\text{trans}} = \ldots$.

A) 1:1  
B) 1:2  
C) 2:1  
D) 1:3  
E) 3:1  

**Solution:**  
For a solid cylinder: $I = \frac{1}{2} m R^2$  
For rolling without slipping: $\omega = v / R$  

$$E_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{1}{2} m v^2$$  
$$E_{\text{trans}} = \frac{1}{2} m v^2 \quad \Rightarrow \quad E_{\text{rot}} / E_{\text{trans}} = \frac{1}{2}$$

34) Two blocks slide on a frictionless surface in the positive $x$ direction. Block 1 has mass $m_1$ and block 2 has mass $m_2 = 3 m_1$. Before the two blocks collide, block 1 has velocity $v_1 = 6 \text{ m/s}$ block 2 has velocity $v_2 = 2 \text{ m/s}$. After the collision, which is elastic and one-dimensional, the velocity of block 1 is ______.

A) $-4 \text{ m/s}$  
B) $-2 \text{ m/s}$  
C) 0  
D) $+2 \text{ m/s}$  
E) $+4 \text{ m/s}$

**Solution:**  
Conservation of linear momentum for collision:  

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \Rightarrow m_1 (6 \text{ m/s}) + 3 m_1 (2 \text{ m/s}) = m_1 v'_1 + 3 m_1 v'_2 \Rightarrow v'_1 + 3 v'_2 = 12 \text{ m/s} \quad (1)$$

For elastic collision:  

$$v_1 - v_2 = -(v'_1 - v'_2) \Rightarrow$$  

$$(v'_1 - v'_2) = -4 \text{ m/s} \quad (2)$$

Solving equations (1) and (2) we have: $v'_1 = 0$ and $v'_2 = 4 \text{ m/s}$.
A red car of mass $m_r = 1500$ kg is moving with a velocity of $v_r = 90.0$ km/h due east and a blue car of mass $m_b = 3000$ kg is moving with a velocity of $v_b = 60.0$ km/h due south. The two cars collide and stick together after the collision. What is the speed of the two car system immediately after the collision?

A) 30 km/h  
B) 50 km/h  
C) 63 km/h  
D) 78 km/h  
E) 130 km/h

**Solution:**

This is a completely inelastic collision. Linear momentum is conserved in the collision as no net external force acts on the two car system.

$$m_r \vec{v}_r + m_b \vec{v}_b = (m_r + m_b) \vec{v} \quad \Rightarrow \quad \vec{v} = \frac{m_r \vec{v}_r + m_b \vec{v}_b}{m_r + m_b}$$

Because $\vec{v}_r \perp \vec{v}_b$, 

$$\vec{v} = \left( \frac{1500 \cdot 90 \hat{i} + 3000 \cdot 60 \hat{j}}{1500 + 3000} \right) \text{km/h} = \left( 30 \hat{i} + 40 \hat{j} \right) \text{km/h} \Rightarrow$$

$$v = \sqrt{(30)^2 + (40)^2} \text{km/h} = 50 \text{km/h}$$

or

$$v = \sqrt{\left( m_r v_r \right)^2 + \left( m_b v_b \right)^2}$$

$$v = \sqrt{\left( 1500 \text{kg} \cdot 90.0 \text{km/h} \right)^2 + \left( 3000 \text{kg} \cdot 60.0 \text{km/h} \right)^2} \frac{1500 \text{kg} + 3000 \text{kg}}{1500 \text{kg} + 3000 \text{kg}} = 50 \text{km/h}$$
36) Four small balls are connected in form of a rectangle by massless rods. The long sides of the rectangle are twice as long as the short sides. The rectangle can rotate around several axes as shown below. Which rotation axis yields the smallest moment of inertia?

A) Axis through 2 balls on long side
B) Axis through centers of short sides
C) Perpendicular to plane of rectangle, through center of rectangle
D) Axis through 2 balls on short side
E) Axis through 2 centers of long sides

Solution:
Let the length of the short side be \( r \), long side \( 2r \), and mass of each ball \( m \), then
\[
I_A = 2mr^2
\]
\[
I_B = 4m\left(\frac{r}{2}\right)^2 = mr^2
\]
\[
I_C = 4m\left(1+\frac{1}{2}\right)r^2 = 5mr^2
\]
\[
I_D = 2m(2r)^2 = 8mr^2
\]
\[
I_E = 4m(r)^2 = 4mr^2
\]

37) A 1.53-kg mass hangs on a rope wrapped around a disk pulley of mass 7.07 kg and radius 66.0 cm. The rope does not slip on the pulley. What is the angular acceleration of the pulley?

A) 4.49 rad/s²
B) 7.98 rad/s²
C) 9.87 rad/s²
D) 12.3 rad/s²
E) zero

Solution:
\[
mg - T = ma \Rightarrow mg = maR + T
\]
\[
TR = I\alpha \Rightarrow T = \frac{1}{2}MaR
\]
\[
mg = (m + \frac{1}{2}M)aR \Rightarrow \alpha = \frac{1}{1 + M/(2m)} \frac{g}{R}
\]
\[
\alpha = \frac{9.80 m/s^2}{1 + 7.07/(2 \cdot 1.53)} \frac{g}{0.660m}
\]
\[
\alpha = 4.49 \text{rad/s}^2
\]
38) A uniform meter stick of mass 0.1 kg is supported by a knife edge at the 50-cm mark and has masses of 0.40 kg and 0.60 kg hanging at the 20-cm and 80-cm marks, respectively. A third mass of 0.30 kg is hung at the ______ mark to keep the stick in balance.

A) 20-cm  
B) 25-cm  
C) 30-cm  
D) 70-cm  
E) 80-cm

**Solution:**
We consider rotation around the point at the 50-cm mark. For balance (static equilibrium) the net torque around this point has to be zero:

\[(0.40 \text{kg})(50 \text{cm} - 20 \text{cm}) + (0.60 \text{kg})(50 \text{cm} - 80 \text{cm}) + (0.30 \text{kg})(50 \text{cm} - x) = 0 \Rightarrow x = 30 \text{cm}\]

39) An aluminum wire 2.0 m in length and 2.0 mm in diameter hangs from the ceiling. How much does the wire stretch when a 10.0-kg mass is hung off it? (The Young's modulus for aluminum is 7.0 \times 10^{10} \text{ N/m}^2.)

A) 0.11 mm  
B) 0.22 mm  
C) 0.33 mm  
D) 0.89 mm  
E) 0.99 mm

**Solution:**
By definition of the Young's modulus:

\[Y = \frac{F}{\Delta L / L} \Rightarrow \Delta L = \frac{LF}{YA} = \frac{Lmg}{Y\pi r^2}\]

\[\Delta L = \frac{(2.0 \text{m})(10.0 \text{kg})(9.8 \text{m/s}^2)}{(7.0 \cdot 10^{10} \text{ N/m}^2)\pi(1.0 \cdot 10^{-3} \text{ m})^2}; \quad \Delta L = 0.89 \cdot 10^{-3} \text{ m} = 0.89 \text{mm}.\]
The situation described below pertains to the next two questions:

A uniform rod of mass 12.0 kg and length 4.00 m is attached at one end with a hinge to a wall. The other end is supported by a massless cable attached to the ceiling. The angle between the ceiling and the cable is 60° (see figure).

40) What is the tension in the cable?
   A) 68 N  
   B) 136 N  
   C) 149 N  
   D) 183 N  
   E) 204 N

Solution:
We consider rotation about the hinge. The static equilibrium condition for the torque:

\[ mgL/2 - TL \sin(60°) = 0 \implies T = \frac{mg}{2 \sin(60°)} = \frac{mg}{\sqrt{3}}; \]

\[ T = \frac{(12.0\, \text{kg})(9.8\, \text{m/s}^2)}{\sqrt{3}}; \quad T = 68\, \text{N} \]

41) The cable suddenly snaps. What is the angular acceleration of the rod as it is released?

   A) 3.68 rad/s^2  
   B) 4.90 rad/s^2  
   C) 6.75 rad/s^2  
   D) 9.24 rad/s^2  
   E) 14.7 rad/s^2

Solution:
Moment of inertia for the rod with respect to the end point:

\[ I = I_{cm} + m(L/2)^2 = \frac{1}{12} mL^2 + \frac{1}{4} mL^2 = \frac{1}{3} mL^2 \]

Second Newton’s law for rotation: \( \tau = I \alpha \implies \alpha = \frac{\tau}{I} \implies \alpha = \frac{mgL/2}{mL^2/3} = \frac{3g}{2L} \quad \alpha = \frac{3 \cdot 9.8\, \text{m/s}^2}{2 \cdot 4\, \text{m}}; \quad \alpha = 3.68 \, \text{rad/s}^2 \]
42) Consider a hollow sphere where the mass $M$ is uniformly concentrated on the surface. If the sphere is of radius $R$, what is the moment of inertia around an axis that is tangent to the surface of the sphere?

A) $(2/3) \, MR^2$
B) $MR^2$
C) $(7/5) \, MR^2$
D) $(5/3) \, MR^2$
E) $2MR^2$

**Solution:**

$$I_{cm} = \frac{2}{3} MR^2; \quad I = I_{cm} + MR^2 \quad \Rightarrow \quad I = \frac{5}{3} MR^2$$

43) A ball of putty with mass $m = 0.4 \, kg$ and speed $v = 5.0 \, m/s$ hits a rod of mass $M = 0.6 \, kg$ and length $L = 4.0 \, m$ that is suspended from the ceiling by a hinge as shown in the figure. The ball hits the rod half-way between the hinge and its bottom end and sticks to the rod after the collision. What is the angular velocity of the ball-rod system after the collision?

A) $0.3 \, rad/s$
B) $0.8 \, rad/s$
C) $1.4 \, rad/s$
D) $1.7 \, rad/s$
E) $2.1 \, rad/s$

**Solution:**

\[I_{rod} = I_{CM} + ML^2 = \frac{1}{12} ML^2 + M \left(\frac{L}{2}\right)^2 = \frac{1}{4} ML^2\]

\[I_{ball} + I_{rod} = m \left(\frac{L}{2}\right)^2 + \frac{1}{3} ML^2 = \left(\frac{1}{2} m + \frac{1}{3} M\right)L^2\]

Use conservation of angular momentum

\[L_i = L_f \quad \Rightarrow \quad mvL/2 = I \omega \quad \Rightarrow \quad \omega = \frac{mvL}{2I} \quad \Rightarrow \quad \omega = \frac{mv}{2 \left(\frac{1}{4} m + \frac{1}{3} M\right)L}\]

\[\omega = \frac{(0.4 \, kg)(5.0 \, m/s)}{2 \left(\frac{1}{4} 0.4 \, kg + \frac{1}{3} 0.6 \, kg \times 4.0 \, m\right)}; \quad \omega = \frac{5}{6} \, rad/s = 0.8 \, rad/s\]
44) A 1-kg gold bar falls off a pirate ship and sinks to the bottom of the ocean, where the pressure is 500 atm. The volume of the gold bar at the bottom of the ocean is ______ than it was on the pirate ship. (The bulk modulus of gold is 180 GPa.)

a. 0.0028% smaller  
b. 0.028% smaller  
c. 0.28% smaller  
d. the same  
e. 0.28% larger

Solution:

\[ B = -\frac{\Delta P}{\Delta V/V} \Rightarrow \frac{\Delta V}{V} = -\frac{\Delta P}{B} ; \frac{\Delta V}{V} = -\frac{(500-1)\cdot1.01\cdot10^5 \text{ Pa}}{180\cdot10^9 \text{ Pa}}; \]

\[ \frac{\Delta V}{V} = -2.8\cdot10^{-4} = -0.028\% \]

45) A solid cylinder with moment of inertia with respect to its center of mass of \( I = 5 \text{ kg m}^2 \) and mass \( m = 10 \text{ kg} \) is pivoted about an axle through its center, O. A rope wrapped around the outer radius \( R_1 = 1.0 \text{ m} \), exerts a force \( F_1 = 5.0 \text{ N} \) to the right. A second rope wrapped around another section of radius \( R_2 = 0.50 \text{ m} \) exerts a force \( F_2 = 6.0 \text{ N} \) downward. How many radians does the cylinder rotate through in the first 5.0 seconds, if it starts from rest?

A) 2.5 rad  
B) 5.0 rad  
C) 7.5 rad  
D) 13 rad  
E) 20 rad

Solution:

Second Newton’s law for rotation:

\[ \tau = I\alpha \Rightarrow \alpha = \tau/I \Rightarrow \alpha = \frac{F_1R_1 - F_2R_2}{I}. \]

\[ \theta = \frac{\alpha^2}{2} \Rightarrow \theta = \frac{(F_1R_1 - F_2R_2)t^2}{2I}. \]

\[ \alpha = \frac{(5.0\text{ N} \cdot 1.0\text{ m} - 6.0\text{ N} \cdot 0.5\text{ m})(5.0\text{ s})^2}{2 \cdot (5 \text{ kg m}^2)} ; \alpha = 5.0\text{ rad}. \]
46) Planet X has one moon which moves in a circular orbit around the planet with an orbit radius of $422 \times 10^3$ km and a period of 1.77 days. What is the mass of Planet X?

A) $1.3 \times 10^{27}$ kg  
B) $1.5 \times 10^{27}$ kg  
C) $1.7 \times 10^{27}$ kg  
D) $1.9 \times 10^{27}$ kg  
E) $2.5 \times 10^{27}$ kg

Solution:

$$T = \frac{2\pi R^{3/2}}{\sqrt{GM}} \Rightarrow M = \frac{4\pi^2 R^3}{T^2G}$$

$$M = \frac{4\pi^2 (4.22 \cdot 10^8 \text{ m})^3}{(1.77 \cdot 24 \cdot 3600 \text{ s})^2 \cdot 6.67 \cdot 10^{-11} \text{ Nm}^2 / \text{kg}^2} ; \quad M = 1.9 \cdot 10^{27} \text{ kg}$$

47) Two planets have the same surface gravity (the same $g$), but planet B has twice the mass of planet A. If planet A has radius $R$, what is the radius of planet B?

A) $R/2$  
B) $R/\sqrt{2}$  
C) $R$  
D) $\sqrt{2}R$  
E) $2R$

Solution:

$$g = G \frac{M}{R^2} \Rightarrow R = \sqrt[3]{\frac{GM}{g}} \Rightarrow \frac{R_B}{R_A} = \sqrt[3]{\frac{M_B}{M_A}} = \sqrt[3]{2} \Rightarrow \quad R_B = \sqrt{2}R_A$$
48) A horizontal disk (Disk 1) of radius $R$ and mass $M_1$ rotates with an angular speed $\omega_0$. Another disk (Disk 2) with the same rotation axis, same radius, mass $M_2 = \frac{1}{4} M_1$ and no initial angular velocity is dropped on top of the first one. After the drop Disk 2 sticks to Disk 1 and they rotate with a common angular speed of $\omega = \underline{\text{ } }$.

A) zero  
B) $\frac{1}{2} \omega_0$  
C) $\frac{2}{3} \omega_0$  
D) $\frac{3}{4} \omega_0$  
E) $\frac{4}{5} \omega_0$

Solution:

$I_{disk} = \frac{1}{2} m R^2$

Use conservation of the angular momentum.

$L_i = L_f \Rightarrow I_1 \omega_0 = (I_1 + I_2) \omega \Rightarrow \omega = \frac{I_1}{I_1 + I_2} \omega_0 = \frac{M_1}{M_1 + M_2} \omega_0 \Rightarrow \omega = \frac{4}{5} \omega_0$

49) A plane, flying horizontally, releases a bomb, which explodes before hitting the ground. Neglecting air resistance, the center of mass of the bomb fragments, just after the explosion

A) does not move.  
B) moves horizontally.  
C) moves vertically.  
D) moves along a parabolic path.  
E) none of the above

Solution:

$\vec{F}_{ext} = m \vec{a}_{cm} \Rightarrow$ the motion of the center of mass does not change after explosions and collisions. The center of mass of the bomb continues to fall freely and move along a parabolic path after the explosion.
50) A 1500-kg car moving at 25 m/s hits an initially uncompressed horizontal spring with spring constant of $2.0 \times 10^6$ N/m. What is the maximum compression of the spring? (Neglect the mass of the spring.)

A) 0.17 m  
B) 0.34 m  
C) 0.51 m  
D) 0.68 m  
E) 0.80 m

**Solution:**
Use conservation of mechanical energy:

$$E_i = E_f \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{k}} ;$$

$$x = (25 \text{ m/s})\sqrt{\frac{1500 \text{ kg}}{(2.0 \cdot 10^6 \text{ N/m})}} \Rightarrow x = 0.68 \text{ m}.$$
The situation described below pertains to the next two questions:

Several forces $F$ are applied to the rods A, B and C below. All forces have equal magnitude. All rods have the same length and uniformly distributed mass.

51) Rank the **net torque** about the center of mass for each of the rods.

A) $\tau_A > \tau_B > \tau_C$
B) $\tau_C > \tau_B > \tau_A$
C) $\tau_B > \tau_C > \tau_A$
D) $\tau_A > \tau_B > \tau_C$
E) $\tau_C = \tau_B > \tau_A$

**Solution:**
Let $2r$ be the length the rod.

$$\tau_A = rF \quad \tau_B = rF(1 + \sin \theta) \quad \tau_C = 0$$

52) Rank the **net force** for each of the rods.

A) $F_A > F_B > F_C$
B) $F_C > F_B > F_A$
C) $F_B > F_A = F_C$
D) $F_B > F_C > F_A$
E) $F_C = F_B > F_A$

**Solution:**

$F_A = F$

$F_B = 2F \cos(\theta_B / 2)$, where $0 < \theta < 90^\circ$ is angle between two forces

$F_C = 2F \cos(\theta_C / 2) = 2F \cos 45^\circ = F\sqrt{2}$. \hspace{1cm} \theta_B < \theta_C \Rightarrow F_B > F_C$
53) Which statement about types of energies, forces and work is TRUE?

A) The kinetic energy of a rigid body depends only on its center of mass speed.
B) Potential energy is always positive.
C) Conservative forces do only positive work.
D) Static friction can only do negative work.
E) Total mechanical energy of an isolated system is conserved if only conservative forces do work.

A) is false: a rigid body can have rotational kinetic energy even when its center of mass is at rest.
B) is false. The absolute value of potential energy is a matter of convention. It can be negative. Physics only depends on changes in potential energy.
C) is false. For example, gravity decelerates an object that is thrown vertically up. The force is anti-parallel to the displacement. Thus, gravity is doing negative work here.
D) is false. Imagine a block on a truck. When the truck accelerates, so does the block. The force that accelerates the block is the static friction between the truck and the block. For this static friction, the force goes in the direction of the displacement and the work done on the block by the static friction is positive.
E) is true.

54) A diver jumps off a board into a pool rotating around his center of mass. At point 3 (see figure) the diver can be described roughly as a disk with radius $R$ and uniform mass distribution. At point 7 the diver can be described roughly as a rod of length $L = 3R$ and uniform mass distribution. At point 3 his angular speed is $\omega$. What is his angular speed at point 7?

A) $\omega/3$
B) $2/3 \omega$
C) $3/4 \omega$
D) $9/16 \omega$
E) $\omega$

Solution:

\[ I_{\text{disk}} = \frac{1}{2} mR^2 \]
\[ I_{\text{rod}} = \frac{1}{12} mL^2 = \frac{1}{12} m(3R)^2 = \frac{3}{4} mR^2 \]

Use conservation of angular momentum:
\[ L_i = L_f \Rightarrow I_{\text{disk}} \omega = I_{\text{rod}} \omega_f \Rightarrow \omega_f = \left( \frac{I_{\text{disk}}}{I_{\text{rod}}} \right) \omega \Rightarrow \omega_f = \frac{2}{3} \omega \]