28) In the figure, a toy racecar of mass \( m \), with very light wheels is released from rest on the loop-the-loop track. If it is released at a height \( 2R \) above the floor, how high is it above the floor when it leaves the track? Neglect any type of friction in this problem.

A) 1.33 R  
B) 2.00 R  
C) 1.50 R  
D) 1.67 R  
E) 1.25 R  

**Solution**

When the car leaves the track, the only force acting on it is its weight. Suppose the position vector of the car (relative to the center of the loop) makes an angle of \( \theta \) with the horizontal, then the radial component of the force is equal to \( mg \sin \theta \) (The decomposition is actually very similar to the case of the incline plane)

\[ mg \sin \theta = \frac{1}{2} mv^2 / R \rightarrow \frac{1}{2} mv^2 = \frac{1}{2} gR \sin \theta = \frac{1}{2} gh \]

where \( h \) is the height above center of loop.

Conservation of energy \( \rightarrow \frac{1}{2} mv^2 + mg(h + R) = mg(2R) \rightarrow 1.5h = R \)

\[ \therefore \text{height above ground} = R + h = 1.67R \]

29) In a collision between two objects having unequal masses, how does the magnitude of the impulse imparted to the lighter object by the heavier one compare with the magnitude of the impulse imparted to the heavier object by the lighter one?

A) The heavier object receives a larger impulse.  
B) The lighter object receives a larger impulse.  
C) The answer depends on the ratio of the speeds.  
D) The answer depends on the ratio of the masses.  
E) Both objects receive the same impulse.  

**Solution**

Total momentum is conserved in a collision, thus

\[ \bar{p}_1 + \bar{p}_2 = \bar{p}_1' + \bar{p}_2' \Rightarrow \bar{p}_1 - \bar{p}_1' = \bar{p}_2 - \bar{p}_2' \Rightarrow \text{Impulse} = \text{Impulse} \]
30) Two stones, one of mass \( m \) and the other of mass \( 2m \), are thrown directly upward with the same velocity at the same time from ground level and feel no air resistance. Which statement about these stones is true?

A) At its highest point, the heavier stone will have twice as much gravitational potential energy as the lighter one because it is twice as heavy.
B) At their highest point, both stones will have the same gravitational potential energy because they reach the same height.
C) Both stones will reach the same height because they initially had the same amount of kinetic energy.
D) The lighter stone will reach its maximum height sooner than the heavier one.
E) The heavier stone will go twice as high as the lighter one because it initially had twice as much kinetic energy.

Solution

The height a stone reaches does not depend on its mass. Since they have the same speed, they get to the same height (though they started with different kinetic energies). At their maximum height the heavier rock has a larger potential energy \( (U = mgh) \). The potential energy at the highest point is equal to the kinetic energy at ground level. Therefore, the rock of mass \( 2m \) has twice the potential energy at the top as the rock of mass \( m \).

\[
U_i + K_i = U_f + K_f \Rightarrow K_i = U_f \Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow h = \frac{v^2}{2g}
\]

\[
K = \frac{1}{2}mv^2 \Rightarrow K_{H, i} = 2K_{L, i}
\]

The time it takes a rock to get to its maximum height is equal to the time it would take the rock to fall to ground when dropped from rest from that height. The time is independent of the mass of the rock.

\[
y - y_0 = -\frac{1}{2}gt^2 \Rightarrow t = \sqrt{2(y_0 - y)/g}
\]
31) An 8.0 gram bullet is shot into a 4.0 kg block, which initially is at rest, on a frictionless horizontal surface. The bullet remains lodged in the block. The block moves into a spring and compresses it by 5.1 cm. The force constant of the spring is 1900 N/m. The initial velocity of the bullet is closest to:

A) 560 m/s  
B) 600 m/s  
C) 580 m/s  
D) 530 m/s  
E) 620 m/s

**Solution**

During the process when bullet gets lodged in the block momentum is conserved.

\[ mV = (m + M)V' \Rightarrow V' = \frac{m}{m + M}V \]

While the block compresses the spring energy is conserved.

\[ \frac{1}{2}(m + M)V'^2 = \frac{1}{2}(m + M)\frac{m^2}{(m + M)^2}V^2 = \frac{1}{2}kx^2 \]

\[ \Rightarrow v = \sqrt{k(m + M)} \frac{x}{m} = \sqrt{kM} \frac{x}{m} = 556 \text{ m/s} \]

32) Block 1 and block 2, each of mass, \( m \) are released from the top of two inclined planes of the same height making 30° and 60° angles with the horizontal direction, respectively. If the coefficient of friction is the same in both cases, which of the blocks is going faster when it reaches the bottom of its respective incline?

A) We must know the actual masses of the blocks to answer.  
B) Both blocks have the same speed at the bottom.  
C) Block 1 is faster.  
D) Block 2 is faster.  
E) There is not enough information to answer the question because we do not know the value of the coefficient of kinetic friction.

**Solution**

Without friction both blocks would have the same speed at the bottom of the incline. Total mechanical energy would be conserved and both blocks started with the same potential energy \( U = mgh \) and no kinetic energy. With friction present the energy lost due to friction is given by the work done by friction on the blocks \( W_{\text{non-conservative}} = -f_kL = -(\mu_k mg \cos \theta) \frac{h}{\sin \theta} = -(\mu_k mg) \frac{h}{\tan \theta} \), where \( f_k \) is the kinetic friction force, \( L \) is the distance the blocks slide done the incline, \( N \) is the normal force, \( mg \) is the weight of the blocks and \( \theta \) is the angle between the incline and the horizontal. For block 1 \( \tan 30^\circ = 0.58 \) and for block 2 \( \tan 60^\circ = 1.73 \). Block 1 loses more energy due to friction and thus has less kinetic energy and speed at the bottom of the incline.
33) In the figure, a graph of potential energy $U$ versus position $x$ for a particle moving in a straight line is shown. For the region shown, it is true that

A) The force on the particle would be strongest when the particle is near the origin.
B) This could not represent an actual physical situation, since the drawing shows the potential energy going negative, which is not physically realizable.
C) There are three positions of stable equilibrium.
D) The force on the particle would be greatest when the particle is near point $D$.
E) For a given value of $x$, the particle can have a total energy that lies either above or below the value given by the curve at that point.

**Solution**
A is correct; the magnitude of the force is proportional to the slope of the $U$ curve. $U$ is steepest near the origin.
B is false; $U$ can be negative.
C is false; three positions with zero slopes but only the two local minima are stable equilibriums.
D is false; local maximum for $U$ means force is zero at $D$.
E is false; total energy is $K+U$ and $K$ is always bigger than or equal to zero, so total energy cannot be below the curve.

34) A 2.0 gram bead slides along a frictionless wire, as shown in the figure. At point $A$, the bead is moving to the right (positive $x$ direction) but with negligible speed. What is the speed of the bead at point $C$?

A) Zero.
B) −4.4 m/s
C) +4.4 m/s
D) +2.0 m/s
E) −5.9 m/s

**Solution**
Since the wire is frictionless, gravity is the only force doing work. Gravity is a conservative force thus the total mechanical energy at points $A$ and $C$ is the same.

\[ E_A = E_C \]
\[ U_A + K_A = U_C + K_C \]
\[ mgh_A + 0 = mgh_c + \frac{1}{2}mv_A^2 \]
\[ \Rightarrow v_A = \sqrt{2g(h_A - h_c)} = 2.0 \text{ m/s} \]
35) A girl of mass 57 kg throws a ball of mass 0.7 kg against a wall. The ball strikes the wall horizontally with a speed of 29 m/s, and it bounces back with this same speed. The ball is in contact with the wall 0.05 s. What is the average force exerted on the wall by the ball?

A) 410 N  
B) 17,000 N  
C) 33,000 N  
D) 66,000 N  
E) 810 N

Solution

initial momentum of ball = $m v = (0.7 \text{ kg})(29 \text{ m/s}) = 20.3 \text{ kg m/s}$

Impulse = $\Delta p = 2mv = F_{av} \Delta t \rightarrow F_{av} = 810 \text{ N}$

36) A firecracker breaks up into several pieces, one of which has a mass of 200 gram and flies off along the positive $x$-axis with a speed of 82.0 m/s. A second piece has a mass of 300 gram and flies off along the positive $y$-axis with a speed of 45.0 m/s. What are the magnitude and direction of the total momentum of these two pieces?

A) 21.2 kg m/s at 56.3° from the positive $x$ axis  
B) 21.2 kg m/s at 39.5° from the positive $x$ axis  
C) 93.5 kg m/s at 28.8° from the positive $x$ axis  
D) 361 kg m/s at 56.3° from the positive $x$ axis  
E) 361 kg m/s at 1.0° from the positive $x$ axis

Solution

\[ \vec{p}_{total} = \vec{p}_1 + \vec{p}_2 \]

\[ = m_1 \vec{v}_1 + m_2 \vec{v}_2 \]

\[ = 0.2\text{kg} \times (82.0\text{m/s},0) + 0.3\text{kg} \times (0,45.0\text{m/s}) \]

\[ = (16.4\text{kg m/s},0) + (13.5\text{kg m/s}) \]

\[ = (16.4,13.5)\text{kg m/s} \]

\[ |\vec{p}_{total}| = \sqrt{(16.4\text{kg m/s})^2 + (13.5\text{kg m/s})^2} = 21.2\text{kg m/s} \]

\[ \phi = \arctan \left( \frac{p_y}{p_x} \right) = \arctan \left( \frac{13.5}{16.4} \right) = 39.5° \]
37) A block of mass \( m = 25 \text{ kg} \) has a speed \( V \) and is behind a block of mass \( M = 64 \text{ kg} \) that has a speed of 0.5 m/s. The surface is frictionless. The blocks collide. After the collision they stick together and have a common speed of 0.9 m/s. The loss of kinetic energy of the blocks due to the collision is closest to:

A) 74 J  
B) 18 J  
C) 2.8 J  
D) 28 J  
E) 13 J

Solution

Total momentum is conserved in the collision.
\[ mV + M(0.5 \text{ m/s}) = (m + M)(0.9 \text{ m/s}) \Rightarrow V = 1.92 \text{ m/s} \]
\[ \Delta KE = \left( \frac{1}{2} mV^2 + \frac{1}{2} M(0.5 \text{ m/s})^2 \right) - \frac{1}{2} (m + M)(0.9 \text{ m/s})^2 = 18.3 \text{ J} \]

38) In the figure, four point masses are placed as shown. The \( x \) and \( y \) coordinates of the center of mass are closest to

A) (2.3 m, 2.8 m)  
B) (2.2 m, 2.6 m)  
C) (2.2 m, 2.7 m)  
D) (2.3 m, 2.6 m)  
E) (2.3 m, 2.7 m)

Solution

\[ x_{CM} = \frac{2.0 \text{ m} \times 2.0 \text{ kg} + 3.0 \text{ m} \times 4.0 \text{ kg} + 5.0 \text{ m} \times 6.0 \text{ kg}}{20.0 \text{ kg}} = 2.3 \text{ m} \]
\[ y_{CM} = \frac{3.0 \text{ m} \times 8.0 \text{ kg} + 5.0 \text{ m} \times 4.0 \text{ kg} + 2.0 \text{ m} \times 6.0 \text{ kg}}{20.0 \text{ kg}} = 2.8 \text{ m} \]
39) A uniform solid sphere has a moment of inertia \( I_T \) about an axis tangent to its surface. The moment of inertia of this sphere about an axis through its center is:

A) \( \frac{2}{7} I_T \)  
B) \( \frac{7}{5} I_T \)  
C) \( \frac{3}{5} I_T \)  
D) \( \frac{2}{5} I_T \)  
E) \( \frac{1}{7} I_T \)

Solution

The moment of inertia of a solid disk around its center is \( I_{CM} = \frac{2}{5} MR^2 \). Using the parallel-axis theorem, we get for the moment of inertia through an axis tangent to the sphere:

\[
I_T = I_{CM} + MR^2 = I_{CM} + \frac{5}{2} I_{CM} = \frac{7}{2} I_{CM} \Rightarrow I_{CM} = \frac{2}{7} I_T
\]

40) A solid disk and a solid sphere are released simultaneously at the top of an inclined plane. They roll down without slipping. Which will reach the bottom first?

A) The one of greatest mass.  
B) The sphere.  
C) The one of smallest diameter.  
D) The disk.  
E) They will reach the bottom at the same time.

Solution

Conservation of mechanical energy gives

\[
mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \Rightarrow v^2 = \frac{2gh}{1 + \frac{I}{(mR^2)}}
\]

The fastest one will be the one with the smallest \( I/(mR^2) \) ratio. Thus the sphere will be faster since \( I_{solid\ sphere} = \frac{2}{5} mR^2 \) and \( I_{solid\ disk} = \frac{1}{2} mR^2 \).
41) A block of mass of $m = 4.0$ kg is attached to a light string, which is wrapped around a spool of radius $R = 0.5$ m and mass of $M = 4.0$ kg. The spool, which can be regarded as a hollow cylinder, is suspended from the ceiling, and the block is then released from rest a distance 4.80 m above the floor. The angular speed of the spool, when the block hits the floor, is closest to

A) 16 rad/s  
B) 14 rad/s  
C) 12 rad/s  
D) 10 rad/s  
E) 8.0 rad/s

Solution

Energy is conserved in the process, thus the sum of kinetic energy of translation and kinetic energy of rotation in the final state is equal to the potential energy in the initial state.

$$E_i = E_f$$
$$U_i + KE_i = U_f + KE_f$$
$$mgh + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{(use v = R\omega and I = mR^2)}$$

$$\rho grh = \frac{1}{2} \rho R^2 \omega^2 + \frac{1}{2} \rho R^2 \omega^2 = \rho R^2 \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{gh}{R}} = 13.7 \text{ rad/s}$$

42) If the net torque on an object is zero, …

A) the object could be accelerating linearly but it could not be rotating.
B) the forces on the object must add up to zero.
C) the object could be both accelerating linearly and rotating.
D) the object cannot be accelerating linearly.
E) the object must remain at rest.

Solution

An object can rotate with constant angular momentum, when no external net torque is acting on it. An object can accelerate linearly, when a net force is acting on it. No net torque is needed for linear acceleration.
43) A light triangular plate $OAB$ lies in the horizontal plane. Three forces, $F_1 = 3.0$ N, $F_2 = 1.0$ N, and $F_3 = 9.0$ N, act on the plate, which can rotate freely about a vertical axis through point $O$. The net torque around point $O$ is closest to

A) 1.4 Nm  
B) 1.1 Nm  
C) 1.8 Nm  
D) 0.8 Nm  
E) 1.6 Nm

Solution

$$
\tau = rF_\perp \sum \tau = -(0.6 \text{ m})(F_1 \cos 30^\circ) + (0.8 \text{ m}) F_2 + (0) F_3 = -0.8 \text{ N m}
$$

44) A 30.0-kg child sits on one end of a long uniform beam having a mass of 20.0 kg, and a 40.0-kg child sits on the other end. The beam balances when a fulcrum is placed below the beam a distance 1.10 m from the 30.0-kg child. How long is the beam?

A) 2.12 m  
B) 2.07 m  
C) 1.98 m  
D) 2.20 m  
E) 1.93 m

Solution

The net torque around the pivot point has to be zero when the beam is in balance. The weights of the two children and the weight of the beam each contribute a torque.

$$
0 = g(30.0\text{kg})(1.10\text{m})+g(20.0\text{kg})(1.10\text{m} - L/2) - g(40.0\text{kg})(L - 1.10\text{m})
$$

$$
0 = 33.0\text{m} + 22.0\text{m} - 10L - 40L + 44.0\text{m}
$$

$$
\Rightarrow L = 1.98\text{m}
$$
The next two questions pertain to the following situation:

A turntable has a radius of 0.8 m and a moment of inertia of 2.0 kg m^2. The turntable is rotating with an angular velocity of 1.5 rad/s about a vertical axis through its center on frictionless bearings. A 0.40 kg ball is projected horizontally toward the turntable axis with a velocity of 3.0 m/s. The ball is caught by a cup-shaped mechanism on the rim of the turntable.

45) The initial angular momentum of the ball about the turntable axis is closest to:

A) 0.3 kg m^2/s
B) 1.0 kg m^2/s
C) 0.8 kg m^2/s
D) 0.5 kg m^2/s
E) zero

Solution

Momentum \( \vec{p} \) passes through axis of rotation → \( \vec{r} \times \vec{p} = 0 \)

46) The fraction of the initial kinetic energy of the system, which is lost during the capture of the ball, is closest to:

A) 0.60
B) 0.45
C) 0.65
D) 0.50
E) 0.55

Solution

The initial KE is \( K_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 4.05 \) J

Final angular momentum of system = initial angular momentum

\( L = I\omega = 3.0 \) kg m^2/s

Final KE \( = \frac{1}{2}I_f\omega_f^2 = \frac{L_f^2}{2I_f} = (3.0 \text{ kg m^2/s})/(2(0.8 + 0.4 \times 0.8^2) \text{ kg m^2}) = 1.99 \) J

Fraction of KE lost =\( (4.05 - 1.99) / 4.05 = 0.51 \)
47) A metal bar is hanging from a hook in the ceiling when a ball that is moving horizontally suddenly strikes it. The ball is covered with glue, so it sticks to the bar. During this collision

A) … the angular momentum of the system (ball and bar) is conserved about the hook because neither the hook nor gravity exerts any torque on this system about the hook.

B) … the angular momentum of the system (ball and bar) is not conserved because the hook exerts a force on the bar.

C) … the angular momentum of the system (ball and bar) is conserved about the hook because only gravity is acting on the system.

D) … both the angular momentum of the system (ball and bar) and its kinetic energy are conserved.

E) … both the linear momentum and the angular momentum of the system (ball and bar) are conserved.

Solution

The only external forces acting on the ball and bar are gravity and the hook. Neither exerts a torque during the collision.

48) A 10-kg uniform ladder, 5.0 m long, is placed against a smooth wall at a height of \( h = 3.1 \) m. The base of the ladder rests on a rough horizontal surface whose coefficient of static friction is 1.80. An 80-kg block is suspended from the top rung of the ladder, very close to the wall. The force exerted on the wall by the ladder is closest to:

A) 830 N

B) 1500 N

C) 1300 N

D) 1100 N

E) 660 N

Solution

The ladder is in equilibrium and thus the net torque about any axis is zero. We calculate the net torque around the point where the ladder touches the floor. With the mass and length of the ladder \( m \) and \( L \), and \( \theta \) the angle between the ladder and the wall, and \( M \) the mass of the block, we get

\[
mg \frac{L}{2} \sin \theta + Mg \sin \theta - NL \cos \theta = 0
\]

\[
\Rightarrow N = \frac{mg}{2} \frac{\sin \theta}{\cos \theta} + Mg \frac{\sin \theta}{\cos \theta} = \left( \frac{m}{2} + M \right) g \tan \theta = 1055 \text{N}
\]
49) Two compressible solids are formed into spheres of the same size. The bulk modulus \( B_2 \) of sphere two is twice as large as the bulk modulus \( B_1 \) of sphere one. You now increase the pressure on both spheres by the same amount. As a result of the increased pressure, how is the change in volume of sphere two (\( \Delta V_2 \)) related to the change in volume of sphere one (\( \Delta V_1 \))?

A) \( \Delta V_2 = \Delta V_1 \)
B) \( \Delta V_2 = 2 \Delta V_1 \)
C) \( \Delta V_2 = \frac{1}{4} \Delta V_1 \)
D) \( \Delta V_2 = \frac{1}{2} \Delta V_1 \)
E) \( \Delta V_2 = 4 \Delta V_1 \)

Solution

\[
B = -\frac{\Delta p}{\Delta V / V_0} \Rightarrow \Delta V = -\frac{\Delta p}{B} V_0
\]

\[
\frac{\Delta V_2}{\Delta V_1} = \left( -\frac{\Delta p}{B_2} V \right) / \left( -\frac{\Delta p}{B_1} V \right) = \frac{B_1}{B_2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}
\]

\( \Rightarrow \Delta V_2 = \frac{\Delta V_1}{2} \)

50) Planet Z-34 has a mass equal to 1/3 that of Earth, a radius equal to 1/3 that of Earth, and an axial spin rate 1/2 that of Earth. With \( g \) representing, as usual, the acceleration due to gravity on the surface of Earth, the acceleration due to gravity on the surface of Z-34 is

A) 3 \( g \)
B) 9 \( g \)
C) 1/3 \( g \)
D) 1/9 \( g \)
E) 6 \( g \)

Solution

\[
g_Z = G \frac{M_Z}{R_Z^2} = G \frac{M_E / 3}{(R_E / 3)^2} = 3G \frac{M_E}{R_E^2} = 3g
\]
51) Two planets having equal masses are in circular orbit around a star. Planet A has a smaller orbital radius than planet B. Which statement about their potential energies $U$, kinetic energies $K$ and total mechanical energies $E$ are true?

A) $K_A < K_B$ and $U_A < U_B$ and $E_A < E_B$
B) $K_A > K_B$ and $U_A < U_B$ and $E_A > E_B$
C) $K_A > K_B$ and $U_A < U_B$ and $E_A < E_B$
D) $E_A = E_B$
E) $K_A > K_B$ and $U_A > U_B$ and $E_A > E_B$

Solution

Since the orbital radius $R_A < R_B$, we have $U = -\frac{GmM}{R} \Rightarrow U_A < U_B$

For a circular orbit, $v = \sqrt{\frac{GM}{R}} \Rightarrow KE = \frac{1}{2} m \frac{GM}{R} \Rightarrow KE_A > KE_B$

For the total energy we have $E = KE + U = \frac{1}{2} \frac{GmM}{R} - \frac{GmM}{R} = -\frac{1}{2} GmM \Rightarrow E_A < E_B$

52) A horizontal disk rotates about a vertical axis through its center. Point $P$ is midway between the center and the rim of the disk, and point $Q$ is on the rim. If the disk turns with constant angular acceleration, which of the following statements is always true?

A) The tangential acceleration of $Q$ is twice as great as the tangential acceleration of $P$.
B) $P$ and $Q$ have the same angular velocity and different angular acceleration.
C) The radial acceleration of $Q$ is half as great as the radial acceleration of $P$.
D) The tangential acceleration of $Q$ is equal in magnitude to its radial acceleration.
E) $P$ and $Q$ have the same tangential acceleration and the same radial acceleration.

Solution

All points on the disk have the same angular acceleration $\alpha$. The tangential acceleration is given by $a_{tan} = R \alpha$. Point Q sits at twice the radius of point P and thus has twice the tangential acceleration. The radial acceleration is $a_{rad} = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$. Thus, point Q has also twice the radial acceleration of point P. Radial and tangential acceleration can be equal, but only for specific values of $\alpha$ and $\omega$, and only temporary for constant angular acceleration.
53) A ball is released from rest on a no-slip surface, as shown in the figure. After reaching its lowest point, the ball begins to rise again, this time on a frictionless surface as shown in the figure. When the ball reaches its maximum height on the frictionless surface, it is …

A) at a greater height as when it was released.
B) at the same height as when it was released.
C) at a lesser height as when it was released.
D) impossible to tell its relative height with respect to the original height without knowing the mass of the ball.
E) impossible to tell its relative height with respect to the original height without knowing the radius of the ball.

Solution

When the ball has rolled to the lowest point, potential energy \( U = mgh \) has been converted to kinetic energy of translation and rotation. When the ball moves up on the other side, it will remain spinning at constant angular speed as there is no friction to decrease the angular speed. Thus, only the kinetic energy of translation is converted back into potential energy and the ball does not get to the same height from which it started.

54) The base of an aluminum block, which is fixed in place, measures 90 cm by 90 cm, and the height of the block is 60 cm. A force, applied to the upper face and parallel to it, produces a shear strain of \( 6.0 \times 10^{-3} \). The shear modulus of aluminum is \( 3.0 \times 10^{10} \) N/m\(^2\). What is the displacement of the upper face in the direction of the applied force?

A) 5.4 mm
B) 3.6 mm
C) 3.0 mm
D) 4.8 mm
E) 4.2 mm

Solution

Shear strain = \( x / h \) \( \Rightarrow \) \( x = \) (shear strain) \( \times h = 0.0060 \times 0.6m = 3.6\text{mm} \)